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A Presentation on Perturbation Modeling for Ocean Sound Propagation

A Paper Presented at the 12th International Congress on Acoustics, 16-18 July 1986, in Halifax, Nova Scotia, Canada

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Preface

The work reported in this document was completed under NUSC Project No. A92045, Dr. Rolf Kasper, Principal Investigator. A portion of this research was completed while two of us (Duston and Verma) participated in the U.S. Navy-ASEE Summer Faculty Research Program at the Naval Underwater Systems Center, New London Laboratory, in conjunction with NUSC Independent Research Project A92045.

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This document is based on a presentation given at the 12th International Congress on Acoustics in Halifax, Nova Scotia, Canada, 16-18 July 1986.

We assume that the speed of sound in the water and the bottom of the ocean is a function of only the depth, and not the range. We also assume that the ocean and its bottom eventually interface with a rigid halfspace. This problem can be solved by the method of normal modes, involving the eigenvalues and eigenfunctions of a depth dependent ordinary differential equation. Since the sound speed in this problem varies only a little from its average value, ~~we~~ exploit the fact that the eigenfunctions and eigenvalues are known when the sound speed is constant. We investigate the changes in these eigenvalues and eigenfunctions that result from changes in the depth dependent sound speed within the ocean and its bottom, using an algebric formulation of the effect

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of the perturbation. Another more recent approach to finding the changes in the eigenvalues and eigenfunctions is a transmutation approach. We show a method of approximating the kernal of an integral transform and use it to find the first order corrections to the eigenvalues and eigenfunctions. Finally we compare the results of these two approaches with the results of classical perturbation theory for the same problem.

PERTURBATION MODELING FOR OCEAN SOUND PROPAGATION

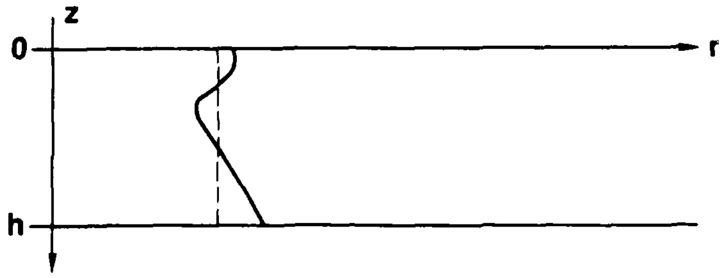
M.D. DUSTON
G.R. VERMA
D.H. WOOD

GRAPH 1

We would like to present an overview of some applications of perturbation theory for ordinary differential equations to ocean sound propagation. We are concerned generally with the problem of calculating the normal modes, and specifically we are involved with finding methods that allow us to find these normal modes in a much more efficient manner than present methods.

We examine three different approaches involving perturbation for the calculation of normal modes.

WE WANT TO SOLVE THE FOLLOWING PROBLEM:



$$p_{rr}(r,z) + \frac{1}{r} p_r(r,z) + p_{zz}(r,z) + k^2 n^2(z)\, p(r,z) = 0,$$

WITH $p(r,0) = 0$ **AND** $p_z(r,h) = 0.$

GRAPH 2

We are dealing at this point with a fairly simplified model of the ocean and make the following assumptions:

1) uniform depth,
2) ocean is an isotropic medium,
3) sound speed a function of depth only,
4) pressure p=0 at the surface,
5) ocean and its bottom are eventually (at some depth h) underlaid by a rigid surface.

Under these assumptions the excess pressure p satisfies the Helmholtz Equation,

$$p_{rr}(r,z) + \frac{1}{r} p_r(r,z) + p_{zz}(r,z) + k^2 n^2(z) p(r,z) = 0,$$

with the boundary conditions

$p(r,0) = 0$ and $p_z(r,h) = 0.$

Here the term $n^2(z)$ reflects the depth dependent nature of the sound speed.

SOLVING BY SEPARATION OF VARIABLES,

$$p(r,z) = \psi(z)\,\theta(r),$$

THE NORMAL MODES MUST SATISFY

$$\psi''(z) + k^2n^2(z)\,\psi(z) = \lambda\psi(z),$$

$$\psi(0) = 0 \quad \text{AND} \quad \psi'(h) = 0$$

GRAPH 3

We solve in a standard manner by using the method of separation of variables $p(r,z) = \psi(z)\Theta(r)$. The depth dependent equation gives the normal modes which satisfy

$$\psi''(z) + k^2n^2(z)\psi(z) = \lambda\psi(z)$$

with the boundary conditions expressed as

$$\psi(0) = 0 \qquad \text{and} \qquad \psi'(h) = 0.$$

THE INDEX OF REFRACTION

$$n^2(z) = 1 + \epsilon s(z)$$

CONTAINS A PERTURBATION $\epsilon s(z)$.

THE CASE $\epsilon = 0$ REPRESENTS AN IDEALIZED OCEAN WITH CONSTANT SOUND SPEED.

GRAPH 4

The quantity $n^2(z)$ is the index of refraction and we represent it as

$$n^2(z) = 1 + \epsilon s(z),$$

where the quantity $\epsilon s(z)$ is considered a perturbation. We introduce the parameter ϵ which reflects the strength of the perturbation.. We assume that the function $s(z)$ is known for the case of interest. When $\epsilon=0$ we have recovered the idealized ocean with constant sound speed.

THE IDEALIZED NORMAL MODE PROBLEM

$$\phi''(z) + k^2\,\phi(z) = \ell\,\phi(z),$$

$$\phi(0) = 0 \quad \text{AND} \quad \phi'(h) = 0$$

WITH L_2 NORMALIZATION HAS SOLUTIONS

$$\phi_m(z) = \sqrt{\frac{h}{2}}\,\sin\frac{(2m-1)\,\pi z}{2h}$$

AND

$$m = 1, 2, 3, \ldots$$

$$\ell_m = k^2 - \left[\frac{(2m-1)\pi}{2h}\right]^2.$$

GRAPH 5

It is well known that for the idealized ocean with constant sound speed that the normal modes satisfy

$$\phi''(z) + k^2\phi(z) = \ell\phi(z)$$

and the boundary conditions we imposed are

$$\phi(0) = 0 \qquad \text{and} \qquad \phi'(h) = 0.$$

It is well known that this problem has a complete set of solutions with eigenfunctions given by

$$\phi_m(z) = \sqrt{\frac{h}{2}}\;\sin\frac{(2m-1)\pi z}{2h} \qquad m = 1, 2, \ldots$$

and corresponding discrete eigenvalues given by

$$\ell_m = k^2 - \left[\frac{(2m-1)\pi}{2h}\right]^2 \qquad m = 1, 2, \ldots .$$

These eigenfunctions have been L_2 normalized.

THE GENERAL PERTURBATION APPROACH

FIND THE EIGENFUNCTIONS ψ_n AND THE EIGENVALUES λ_n OF THE PERTURBED PROBLEM IN TERMS OF THE EIGENFUNCTIONS ϕ_n AND EIGENVALUES ℓ_n OF THE IDEALIZED PROBLEM AND THE PERTURBATION $\epsilon s(z)$.

GRAPH 6

In a perturbation approach we find the eigenfunctions ψ_m and eigenvalues λ_m of the perturbed (depth dependent) in terms of the eigenfunctions ϕ_m and eigenvalues ℓ_m of an idealized problem (which we know) and the perturbation $\epsilon s(z)$ (which we also know). Specifically we look for the changes or corrections which must be made to the idealized eigenfunctions and eigenvalues.

THE CLASSICAL APPROACH

EXPAND IN POWER SERIES IN ϵ

$$\lambda_m = \ell_m + \epsilon\lambda_m^{(1)} + \epsilon^2\lambda_m^{(2)} + \ldots$$

AND

$$\psi_m(z) = \phi_m(z) + \epsilon\psi_m^{(1)}(z) + \epsilon^2\psi_m^{(2)}(z) + \ldots,$$

BUT OBTAIN EACH TERM OF $\psi_m(z)$ ONLY AS A FOURIER SERIES

$$\psi_m^{(i)}(z) = \sum_{p=1}^{\infty} \alpha_{mp}^{(i)} \phi_p(z).$$

GRAPH 7

The first approach we examine is the classical perturbation approach found in Titchmarsh. The eigenvalues and eigenfunctions of the perturbed problem are expended in power series of the parameter ϵ.

The perturbed eigenvalue equals the idealized eigenvalue plus corrections,

$$\lambda_m = \ell_m + \epsilon\lambda^{(1)} + \epsilon^2\lambda^{(2)} + \ldots .$$

The perturbed eigenfunctions equals the idealized eigenfunctions plus corrections,

$$\psi_m(z) = \phi_m(z) + \epsilon\psi_m^{(1)}(z) + \epsilon^2\psi_m^{(2)}(z) + \ldots .$$

However, we find the corrections to the perturbed eigenfunctions only as a Fourier series of the idealized eigenfunctions and we must find the fourier coefficients α in terms of the idealized eigenfunction, the idealized eigenvalues and the perturbation.

THE CLASSICAL RESULTS

THE CORRECTIONS TO FIRST ORDER IN ϵ ARE

$$\lambda_m^{(1)} = \frac{2k^2}{h} \int_0^h s(z) \left[\sin \frac{(2m-1)\pi z}{2h}\right]^2 dz.$$

THE FOURIER COEFFICIENTS ARE

$$\alpha_{mn}^{(1)} = \frac{2k^2}{h} \int_0^h s(z) \sin \frac{(2m-1)\pi z}{2h} \sin \frac{(2n-1)\pi z}{2h} dz,$$

AND

$$\alpha_{mm}^{(1)} = 0.$$

GRAPH 8

In a fairly straight forward manner the explicit equations for the corrections may be derived. We exhibit the corrections to the first order in ϵ. The first order corection to the m^{th} eigenvalue is given by the formula

$$\lambda^{(1)} = \frac{2k^2}{h} \int_0^h s(z) \left[\sin\frac{(2m-1)\pi z}{2h}\right]^2 dz .$$

The first order correction to the eigenfunctions is given in terms of an infinite fourier series whose coefficients are given by

$$\alpha_{mn}^{(1)} = \frac{2k^2}{h} \int_0^h s(z) \sin\frac{(2m-1)\pi z}{2h} \sin\frac{(2n-1)\pi z}{2h} dz \quad \text{and} \quad \alpha_{mm}^{(1)} = 0$$

THE GALERKIN APPROACH

DEFINE MATRICES

$$L = \begin{bmatrix} \ell_1 & & & 0 \\ & \ell_2 & & \\ & & \ell_3 & \\ 0 & & & \ddots \end{bmatrix}, \qquad \Phi = \begin{bmatrix} \phi_1(z) \\ \phi_2(z) \\ \phi_3(z) \\ \vdots \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \lambda_3 & \\ 0 & & & \ddots \end{bmatrix}, \qquad \text{AND } \Psi = \begin{bmatrix} \psi_1(z) \\ \psi_2(z) \\ \psi_3(z) \\ \vdots \end{bmatrix}.$$

GRAPH 9

The next approach we examine is a Galerkin type approach. For compactness of notation we define L an infinte diagonal matrix whose entries are the eigenvalues of the idealized problem, $\Phi(z)$ an infinite column vector whose entries are the eigenfunctions of the idealized problem, Λ an infinite diagonal matrix whose entries are the eigenvalues of the perturbrd problem and $\Psi(z)$ an infinite column vector whose entries are the eigenfunctions of the perturbed problem.

THE GALERKIN APPROACH (Cont'd)

THE PERTURBED PROBLEM IS

$$\Psi'' + k^2 (1 + \epsilon s(z)) \Psi = \Lambda \Psi,$$

$$\Psi(0) = 0 \text{ AND } \Psi'(h) = 0.$$

THE IDEALIZED PROBLEM IS

$$\Phi'' + k^2 \Phi = L\Phi,$$

$$\Phi(0) = 0 \text{ AND } \Phi'(h) = 0.$$

THE GALERKIN APPROACH FINDS A MATRIX D SUCH THAT

$$\Psi(z) = D\Phi(z).$$

GRAPH 10

We may now express the perturbed problem in terms of the vector equation

$$\Psi'' + k^2(1 + \epsilon s(z))\Psi = \Lambda\Psi$$

where $k^2(1 + \epsilon s(z))$ is a scalar quantity and the boundary conditions are

$\Psi(0) = 0$ and $\Psi'(h) = 0$,

where the 0 is a zero vector. The idealized problem is represented by a corresponding vector equation.

In the Galerkin approach we look for a constant linear transformation (constant matrix) that satisfies

$$\Psi = D\Phi$$

In fact, we can guarantee the existence of such a matrix. The entries of the mth row of the matrix D are the coefficients of the m^{th} eigenfunction ψ expressed as a Fourier series in the eigenfunctions of the idealized problem.

THE GALERKIN APPROACH (Cont'd)

WE OBTAIN AN INFINITE ALGEBRAIC EIGENPROBLEM FOR THE MATRIX D^t,

$$(L + \epsilon A)\, D^t = D^t \Lambda,$$

WHERE

$$A = \int_0^h k^2\, s(z)\, \Phi(z)\, \Phi^t(z)\, dz.$$

GRAPH 11

Substituting the tranformation with a normalization constraint we simplify and obtain an infinite algebraic eigenproblem

$$(L + \epsilon A)\, D^t = D^t\, \Lambda,$$

where A is a matrix defined by

$$A = \int_0^h k^2\; s(z)\; \Phi\Phi^t\; dz$$

and ^{t} denotes the transpose. This is really a standard algebraic eigenproblem of the type $MX = \lambda X$ where the columns of D^t are taken to be the vectors X. It is important that Λ multiply D^t on the right so that the m^{th} column of D^t is multiplied by the m^{th} eigenvalue in Λ. In this approach it is not necessary that the parameter ϵ be small. The matrix A has special structure (it is symmetric, and the sum of Hankel and Toeplitz matrices) and it is this special structure which can be exploited to solve the problem more efficiently than an arbitrary eigenproblem.

THE TRANSMUTATION APPROACH

WE EXPRESS THE PERTURBED NORMAL MODES AS

$$\psi_n(z) = \phi_n(z) + \int_h^z K(z,s)\,\phi_n(s)\,ds.$$

EXPAND THE PERTURBLED PROBLEM IN POWER SERIES IN ϵ.

GRAPH 12

The last approach we examine is a transmutation approach. First we express the solution of the perturbed normal mode problem in terms of the integral transform of the solutions of an idealized normal mode problem. We use a transform of the type given by

$$\psi_m(z) - \phi_n(z) + \int_h^z K(z,s)\phi(s)\,ds$$

where we must specify the kernel function $K(z,s)$. In fact, we do not solve for this kernel analytically but instead expand in a power series in the parameter ϵ. We have developed an explicit method to find the coeffient of the power series expansion of the kernel and thus can approximate the kernel function.

THE TRANSMUTATION RESULTS

THE CORRECTIONS TO FIRST ORDER IN ϵ ARE

$$\lambda_n = \ell_n + \epsilon \left| \frac{(2n-1)\pi}{2h} \sqrt{\frac{2}{h}}\, \phi_n^{(1)}(0) \right|$$

AND

$$\psi_n(z) = \phi_n(z) + \epsilon \left| \phi_n^{(1)}(0) \left(\left| 1 - \frac{z}{h} \right| \cos \frac{(2n-1)\pi z}{2h} - \int_0^h \left| 1 - \frac{z}{h} \right| \cos \frac{(2n-1)\pi z}{2h}\, \theta_n^{(0)}(z)\, dz \right) \right.$$

$$\left. + \left(\int_h^z K^{(1)}(z,s)\theta_n^{(0)}(s)\, ds - \int_0^h \left| \int_h^z K^{(1)}(z,s)\theta_n^{(0)}(s)\, ds \right| \theta_n^{(0)}(z)\, dz \right) \right| .$$

GRAPH 13

We can express the perturbed eigenvalues to the first order in ϵ as

$$\lambda_n = \ell_n + \epsilon \left[\frac{(2n-1)\pi}{2h} \sqrt{\frac{2}{h}}\ \phi_n^{(1)}(0) \right]$$

and the perturbed eigenfunctions as

$$\psi_n(z) = \phi_n(z)$$
$$+ \epsilon \left\{ \phi_n^{(1)}(0) \left(\left[1 - \frac{z}{h} \right] \cos \frac{(2n-1)\pi z}{2h} - \int_0^h \left[1 - \frac{z}{h} \right] \cos \frac{(2n-1)\pi z}{2h}\ \theta_n^{(0)}(z)\ dz \right] \right.$$
$$\left. + \left(\int_h^z K^{(1)}(z,s)\theta_n^{(0)}(s)\ ds - \int_0^h \left[\int_h^z K^{(1)}(z,s)\theta_n^{(0)}(s)\ ds \right] \theta_n^{(0)}(z)\ dz \right] \right\} .$$

While the correction to the eigenfunction may look complicated it is the whole correction. We have an explicit formula without having to resort to Fourier series expansion.

THE TRANSMUTATION RESULTS (Cont'd)

THE CORRECTIONS TO FIRST ORDER IN ϵ ARE DEFINED IN TERMS OF

$$\phi_n^{(1)}(0) = \int_0^h K^{(1)}(0,s)\,\theta_n^{(0)}(s)\,ds$$

AND

$$\epsilon\, K^{(1)}(z,s) = -\frac{k^2}{2}\left(\int_h^{\frac{z+s}{2}} [n^2(\zeta) - 1]\,d\zeta + \int_h^{\frac{z-s+2h}{2}} [n^2(\zeta) - 1]\,d\zeta\right).$$

GRAPH 14

The corrections for the eigenvalues and eigenfunctions are expressed in terms of $\phi^{(1)}(0)$ and $K^{(1)}(z,s)$. The first constant is defined by the expression

$$\phi_n^{(1)}(0) = \int_0^h K^{(1)}(0,s)\theta_n^{(0)}(s)\,ds,$$

and the first order in epsilon term of the power series expansion of the kernel is given by

$$\epsilon K^{(1)}(z,s) = -\frac{k^2}{h}\left(\int_h^{(z+s)/2} [n^2(\zeta) - 1]\,d\zeta + \int_h^{(z-s)/2 + h} [n^2(\zeta) - 1]\,d\zeta\right)$$

We again have the perturbation type result where the corrections are explicit functions of θ, the unperturbed eigenfunction, ℓ, the unperturbed eigenvalue and the expression $[n^2(z) - 1]$, which is the perturbation.

COMPARISON OF APPROACHES

	POWER SERIES IN ϵ	FOURIER SERIES	NOT EXPLICIT FORMULAS
CLASSICAL	✓	✓	
GALERKIN		✓	✓
TRANSMUTATION	✓		

GRAPH 15

In summary let us examine each approach. The classic approach gives explicit formulas that are readily obtained for the corrections but depends on both power series expansion in ϵ and infinite Fourier series expansion of the corrections for the eigenfunctions. The Galerkin approach is neither dependent on small ϵ nor does it require a power series expansion in that parameter, however, the eigenfunctions are given in terms of a Fourier series and does not give explicit formulas for the corrections. Finally the transmutation method gives explicit formulas without the need for the Fourier series expansion, but it still requires a power series expansion in ϵ and the calculations required are somewhat more involved.

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